Pre-Algebra
Student Workbook

Name:________________________

Period:___
1A: Graphic Organizer

Study Skill  Take a few minutes to explore the general contents of this text. Begin by looking at the cover. What does the cover art say about mathematics? What do the pages before the first page of Chapter 1 tell you? What special pages are in the back of the book to help you?

Write your answers. Use the Table of Contents page for this chapter at the front of the book.

1. What is the title of this chapter?  

2. Name four topics that you will study in this chapter.
   ______________________________  ______________________________
   ______________________________  ______________________________

3. What is the topic of the Problem Solving lesson?  

4. Complete the graphic organizer below as you work through the chapter.
   1. Write the title of the chapter in the center oval.
   2. When you begin a lesson, write the name of the lesson in a rectangle.
   3. When you complete that lesson, write a skill or key concept from that lesson in the outer oval linked to that rectangle.
      Continue with steps 2 and 3 clockwise around the graphic organizer.

   [Graphic Organizer Diagram]
Lesson 1-1

Variables and Expressions

Lesson Objectives

- Identify variables, numerical expressions, and variable expressions
- Write variable expressions for word phrases

NAEP 2005 Strand: Algebra
Topic: Variables, Expressions, and Operations
Local Standards: 

Vocabulary

A variable is _________________________________

A variable expression is _________________________________

\[ \text{variable} \rightarrow m \quad \leftarrow \text{miles on 10 gallons} \]

\[ \text{variable expression} \rightarrow m \div 10 \quad \leftarrow \text{miles per gallon} \]

Examples

1. Identifying Expressions
   Identify each expression as a numerical expression or a variable expression. For a variable expression, name the variable.
   a. \[ 7 \times 3 \]
      expression
   
   b. \[ 4t \]
      expression
      \( t \) is the variable.

2. Writing Variable Expressions
   Write a variable expression for the cost of \( p \) pens priced at 29¢ each.

   Words
   \[ \text{29¢ times number of pens} \]
   Let \( p \) = number of pens.

   Expression
   \[ \square \cdot \square \]

   The variable expression \[ \square \] or \[ \square \] describes the cost of \( p \) pens.
Quick Check

1. Identify each expression as a *numerical expression* or a *variable expression*. For a variable expression, name the variable.
   a. $8 \div x$  
   b. $100 \times 6$  
   c. $d + 43 - 9$

2. a. Bagels cost $.50 each. Write a variable expression for the cost of $b$ bagels.

   b. **Measurement** Write a variable expression for the number of hours in $m$ minutes.

3. Write a variable expression for each word phrase.

<table>
<thead>
<tr>
<th>Word Phrase</th>
<th>Variable Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nine more than a number $y$</td>
<td></td>
</tr>
<tr>
<td>4 less than a number $n$</td>
<td></td>
</tr>
<tr>
<td>A number $z$ times three</td>
<td></td>
</tr>
<tr>
<td>A number $a$ divided by 12</td>
<td></td>
</tr>
<tr>
<td>5 times the quantity 4 plus</td>
<td></td>
</tr>
<tr>
<td>a number $c$</td>
<td></td>
</tr>
</tbody>
</table>
1-1 • Guided Problem Solving

GPS Student Page 6, Exercise 33

Mia has $20 less than Brandi. Brandi has \( d \) dollars. Write a variable expression for the amount of money Mia has.

**Understand the Problem**

1. Who has more money, Brandi or Mia? _________________

2. What operation do you think of when you hear the phrase *less than*? _________________

3. Describe how much money Mia has, compared to how much Brandi has. _________________

4. What does the variable \( d \) represent? _________________

5. The problem asks you to write an expression for what? _________________

**Make and Carry Out a Plan**

6. You are given two pieces of information in the problem: the amount of money that Brandi has and the fact that Mia has $20 less than Brandi.
   To write an expression for the amount Mia has, start by writing the amount that Brandi has. _________________

7. To complete the expression, show the subtraction of $20 from the amount that Brandi has. _________________

**Check the Answer**

8. You know that Brandi has $20 more than Mia. To check that your expression for this amount is correct, add $20 to it to see whether you get Brandi’s amount.

   _________________

**Solve Another Problem**

9. Deena has 5 more marbles than Jonna. Jonna has \( m \) marbles.
   Write an expression to represent the number of marbles Deena has. _________________
Write an expression for each quantity.
1. the value in cents of 5 quarters
2. the value in cents of \( q \) quarters
3. the number of months in 7 years
4. the number of months in \( y \) years
5. the number of gallons in 21 quarts
6. the number of gallons in \( q \) quarts

Write a variable expression for each word phrase.
7. \( 9 \) less than \( k \)
8. \( m \) divided by 6
9. twice \( x \)
10. 4 more than twice \( x \)
11. the sum of eighteen and \( b \)
12. three times the quantity 2 plus \( a \)

Tell whether each expression is a numerical expression or a variable expression. For a variable expression, name the variable.
13. \( 4d \)
14. \( 74 + 8 \)
15. \( \frac{4(9)}{6} \)
16. \( 14 - p \)
17. \( 5k - 9 \)
18. \( 3 + 3 + 3 + 3 \)
19. \( 19 + 3(12) \)
20. \( 25 - 9 + x \)

The room temperature is \( c \) degrees centigrade. Write a word phrase for each expression.
21. \( c + 15 \)
22. \( c - 7 \)
Reteaching 1-1

Variables and Expressions

A variable is a letter that stands for a number.
Thomas needs $2 to ride the bus to Videoland. How much can he spend on video games for each amount in the table?

<table>
<thead>
<tr>
<th>Thomas Has</th>
<th>Thomas Can Spend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expression</td>
<td>Amount</td>
</tr>
<tr>
<td>$5</td>
<td>$3</td>
</tr>
<tr>
<td>$7</td>
<td>$5</td>
</tr>
<tr>
<td>$10</td>
<td>$8</td>
</tr>
<tr>
<td>$d</td>
<td>$d - 2</td>
</tr>
</tbody>
</table>

The letter $d$ is a variable that stands for the amount of money Thomas has. The expression $d - 2$ is a variable expression. It has a variable ($d$), a numeral (2), and an operation symbol ($-$).

Videoland tokens cost one dollar for 4. How many tokens can Jennifer buy for each amount of money in the table?

<table>
<thead>
<tr>
<th>Jennifer Has</th>
<th>Tokens Jennifer Can Buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expression</td>
<td>Amount</td>
</tr>
<tr>
<td>1. $5</td>
<td></td>
</tr>
<tr>
<td>2. $8</td>
<td></td>
</tr>
<tr>
<td>3. $6</td>
<td></td>
</tr>
<tr>
<td>4. $d dollars</td>
<td></td>
</tr>
</tbody>
</table>

Write a variable expression for each word phrase.

5. $h$ divided by 7
6. $j$ decreased by 9
7. twice $x$
8. two more than $y$
9. the quotient of 42 and a number $s$
10. the product of a number $d$ and 16
A Variable Maze

To find your way through the maze, look for the word phrase that corresponds to each numbered junction. Move in the direction of the letter (A or B) beside the correct variable expression.

1. the product of \(m\) and \(n\)  
   \[\text{A. } mn\]  
   \[\text{B. } m + n\]

2. two less than \(k\)  
   \[\text{A. } 2 - k\]  
   \[\text{B. } k - 2\]

3. the sum of \(a\) and \(b\)  
   \[\text{A. } a + b\]  
   \[\text{B. } ab\]

4. \(h\) increased by 10  
   \[\text{A. } h + 10\]  
   \[\text{B. } h(10)\]

5. the quotient of \(x\) and 4  
   \[\text{A. } x ÷ 4\]  
   \[\text{B. } x(4)\]

6. eight more than \(p\)  
   \[\text{A. } 8(p)\]  
   \[\text{B. } p + 8\]

7. the difference of \(y\) and 3  
   \[\text{A. } y - 3\]  
   \[\text{B. } y ÷ 3\]

8. twenty decreased by \(c\)  
   \[\text{A. } 20 - c\]  
   \[\text{B. } 20 + c\]

9. twice the difference of \(h\) and 5  
   \[\text{A. } 2h - 5\]  
   \[\text{B. } 2(h - 5)\]

10. \(t\) less than two  
    \[\text{A. } t - 2\]  
    \[\text{B. } 2 - t\]

11. \(k\) increased by twice \(x\)  
    \[\text{A. } k + 2x\]  
    \[\text{B. } k + 2 + x\]

12. \(s\) divided by \(n\)  
    \[\text{A. } n ÷ s\]  
    \[\text{B. } s ÷ n\]

13. \(y\) times the sum of 2 and \(l\)  
    \[\text{A. } y(2 + l)\]  
    \[\text{B. } (y + 2)l\]
Lesson 1-2

The Order of Operations

Lesson Objectives
- Use the order of operations
- Use grouping symbols

NAEP 2005 Strand: Number Properties and Operations
Topic: Properties of Number and Operations
Local Standards: ________________________________

Key Concepts

Order of Operations
1. Work inside _________ symbols.
2. _________ and _________ in order from left to right.
3. _________ and _________ in order from left to right.

Examples

1. Simplifying Expressions
   Simplify $8 - 2 \cdot 2$.
   
   First multiply.
   
   Then subtract.

2. Using the Order of Operations
   Simplify $12 \div 3 - 1 \cdot 2 + 1$.
   
   Multiply and divide from left to right.
   
   Add and subtract from left to right.
   
   Add.

Quick Check

1. Simplify each expression.
   
   a. $2 + 5 \times 3$
   
   b. $12 \div 3 - 1$
   
   c. $10 - 1 \cdot 7$
Simplifying With Grouping Symbols

Simplify \(20 - 3[(5 + 2) - 1]\).

Add within parentheses.

Subtract within brackets.

Multiply.

Subtract.

Quick Check

2. Simplify each expression.
   a. \(4 - 1 \cdot 2 + 6 \div 3\)

b. \(5 + 6 \cdot 4 \div 3 - 1\)

3. Simplify each expression.
   a. \(2((13 - 4) \div 3]\)

b. \(1 + \frac{10 - 2}{4}\)
On the Job  A part-time employee worked 4 hours on Monday and 7 hours each day for the next 3 days. Write and simplify an expression that shows the total number of hours worked.

Understand the Problem

1. How many hours did the employee work on Monday? ____________

2. How many hours did the employee work each day on Tuesday, Wednesday, and Thursday? ____________

3. For how many days did the employee work 7 hours? ____________

Make and Carry Out a Plan

4. What operation should you use to find the total number of hours worked in the 3 days that the employee worked 7 hours each day? ________________

5. Write an expression to find the total number of hours worked during those 3 days. ________________

6. What operation should you use to combine the hours the employee worked on Monday with the hours worked in the next 3 days? ________________

7. Write an expression for the total number of hours worked on Monday and the number of hours worked in the next 3 days. ________________

8. Simplify the expression you wrote for Step 7 to find the total number of hours worked. Remember to use the correct order of operations. ________________

Check the Answer

9. You can check your work by writing an expression that adds the number of hours worked on each day. This expression should simplify to the number of hours you found in Step 8 above.

______________________________

Solve Another Problem

10. Carter bought 4 books for $8 each and another book for $5. Write and simplify an expression to find the total cost of the books he bought.

______________________________
Practice 1-2

*The Order of Operations*

**Simplify each expression.**

1. \(3 + 15 - 5 \cdot 2\) __________
2. \(5 \cdot 6 + 2 \cdot 4\) __________
3. \(48 \div 8 - 1\) __________
4. \(68 - 12 \div 2 \div 3\) __________
5. \(6(2 + 7)\) __________
6. \(25 - (6 \cdot 4)\) __________
7. \(3[9 - (6 - 3)] - 10\) __________
8. \(60 \div (3 + 12)\) __________
9. \(4 - 2 + 6 \cdot 2\) __________
10. \(18 \div (5 - 2)\) __________
11. \(\frac{16 + 24}{30 - 22}\) __________
12. \(2[4(9 - 7) + 1]\) __________
13. \((8 \div 8 + 2 + 11) \div 2\) __________
14. \(9 + 3 \cdot 4\) __________
15. \(18 \div 3 \cdot 5 - 4\) __________
16. \(10 + 28 \div 14 - 5\) __________

**Insert grouping symbols to make each number sentence true.**

17. \(3 + 5 \cdot 8 = 64\)
18. \(4 \cdot 6 - 2 + 7 = 23\)
19. \(10 \div 3 + 2 \cdot 4 = 8\)
20. \(3 + 6 \cdot 2 = 18\)

A city park has two walkways with a grassy area in the center, as shown in the diagram.

21. Write an expression for the area of the sidewalks, using subtraction.

_____________________________________________________________________

22. Write an expression for the area of the sidewalks, using addition.

_____________________________________________________________________

Compare. Use >, <, or = to complete each statement.

23. \((24 - 8) \div 4\) \(\square\) \(24 - 8 \div 4\)
24. \(3 \cdot (4 - 2) \cdot 5\) \(\square\) \(3 \cdot 4 - 2 \cdot 5\)
25. \((22 + 8) \div 2\) \(\square\) \(22 + 8 \div 2\)
26. \(20 \div 2 + 8 \cdot 2\) \(\square\) \(20 \div (2 + 8) \cdot 2\)
27. \(11 \cdot 4 - 2\) \(\square\) \(11 \cdot (4 - 2)\)
28. \((7 \cdot 3) - (4 \cdot 2)\) \(\square\) \(7 \cdot 3 - 4 \cdot 2\)
Reteaching 1-2

The Order of Operations

Simplify $\frac{18 + 4}{2} - 3(10 \cdot 2 - 3 \cdot 6)$

Work inside grouping symbols first.

A fraction bar is a grouping symbol.

Divide the fraction.

Multiply within the parentheses.

Subtract within the parentheses.

Multiply.

Simplify each expression.

1. $8 + 2 \times 7$

2. $16 \div 2 - 5$

3. $\frac{8 + 12}{5}$

4. $4 - 24 \div 8$

5. $3 + 2 \cdot 5 - 4$

6. $15 - 2(5 - 2)$

7. $9 \cdot 3 + 2 \cdot 5$

8. $12 \div 4 - 6 \div 3$

9. $5(2 + 4) + 15 \div (9 - 6)$

10. $3 \cdot 2 + 16 \div 4 - 3$

11. $(18 + 7) \div (3 + 2)$

12. $3[8 - 3 \cdot 2 + 4(5 - 2)]$

13. $4 \cdot 9 + 8 \div 2 - 6 \cdot 5$

14. $[7 + 3 \cdot 2 + 8] \div 7$

15. $53 - [3(8 + 2) + 5(9 - 5)]$

16. $(20 + 22) \div 6 + 1$

17. $2[9(6 - 5)]$

18. $5 + 3 \cdot 4 - 8 + 2 \cdot 7$
Enrichment 1-2

Binary Operations

A binary operation is an operation performed on two numbers. Addition, subtraction, multiplication, and division are all binary operations. Once you known how to use a binary operation, you can perform it on any two numbers.

Here is a new binary operation. # means “multiply the numbers, then add the second number to the product.”

Example 5 # 4 = 5 \times 4 + 4 = 24

Use the operation # to solve.

1. 3 # 2  
2. 8 # 5  
3. 1 # 7  
4. 10 # 9  
5. 4 # (2 # 7)  
6. (3 # 4) # 5

7. Evaluate 3 # 5 + 2 doing the operation # first. 
8. Evaluate 3 # 5 + 2 doing the operation + first. 
9. Complete the following order of operation rule, guaranteeing that the value of 3 # 5 + 2 will be 28: When evaluating an expression involving # and +,

10. Use your rule to evaluate 8 # 6 + 3.

Discover how to use each binary operation by studying the examples. Then perform the operation on the given numbers.

Example 2 \times 5 = 20  \quad 4 \times 3 = 24  \quad 7 \times 5 = 70  \quad 10 \times 10 = 200

11. 3 \times 6  
12. 5 \times 5  
13. 8 \times 2  
14. 12 \times 12  
15. (2 \times 2) \times 4  
16. 2 \times (2 \times 4)

Example 4 \div 1 = 17  \quad 6 \div 3 = 39  \quad 9 \div 2 = 83  \quad 10 \div 1 = 101

17. 5 \div 3  
18. 2 \div 7  
19. 7 \div 13  
20. 12 \div 10  
21. (2 \div 3) \div 4  
22. 2 \div (3 \div 4)
Lesson 1-3

Lesson Objectives

- Evaluate variable expressions
- Solve problems by evaluating expressions

NAEP 2005 Strand: Algebra
Topic: Variables, Expressions, and Operations
Local Standards: __________________________

Vocabulary

To evaluate an expression is __________________________

Examples

1. Evaluating a Variable Expression

Evaluate 18 + 2g for g = 3.

\[ 18 + 2g = 18 + 2(3) \]

Replace the variable.

\[ = 18 + \square \]

Multiply.

\[ = \square \]

Add.

2. Replacing More Than One Variable

Evaluate \(2ab - \frac{c}{3}\) for \(a = 3, b = 4,\) and \(c = 9.\)

\[ 2ab - \frac{c}{3} = 2 \cdot \square \cdot \square - \square \]

Replace the variables.

\[ = 2 \cdot 3 \cdot 4 - \square \]

Work within grouping symbols.

\[ = \square \cdot 4 - 3 \]

Multiply from left to right.

\[ = \square - 3 \]

Multiply.

\[ = \square \]

Subtract.

Quick Check

1. Evaluate each expression.
   a. \(63 - 5x,\) for \(x = 7\)
   
   b. \(4(t + 3) + 1,\) for \(t = 8\)

   c. \(6(g + h),\) for \(g = 8\) and \(h = 7\)

   d. \(2xy - z,\) for \(x = 4, y = 3,\) and \(z = 1\)

   e. \(\frac{r + s}{2},\) for \(r = 13\) and \(s = 11\)
Examples

3. The Omelet Café buys cartons of 36 eggs.
   a. Write a variable expression for the number of cartons the café should buy for $x$ eggs.
      
      An expression for $x$ eggs is \[ \frac{x}{36} \].
   
   b. Evaluate the expression for 180 eggs.
      
      \[ \frac{x}{36} = \frac{180}{36} \]  
      \[ \text{Evaluate for } x = 180. \]
      
      \[ \frac{x}{36} = \frac{180}{36} = 5 \]
      
      Divide.

      The Omelet Café should buy 5 cartons to get 180 eggs.

4. The One Pizza restaurant makes only one kind of pizza, which costs $16. The delivery charge is $2. Write a variable expression for the cost of having $p$ pizzas delivered. Evaluate the expression to find the cost of having five pizzas delivered.

   \[ \text{Expression} \quad 16 \cdot p + 2 \]

   Evaluate the expression for $p = 5$

   \[ 16 \cdot 5 + 2 = 82 \]
   \[ = 82 \]
   
   It costs $82 to have five pizzas delivered.

Quick Check

2. The café in Example 3 pays $21 for each case of bottled water. Write a variable expression for the cost of $c$ cases. Evaluate the expression to find the cost of 5 cases.

3. Evaluate the expression in Example 4 to find the cost of ordering 8 pizzas.
1-3 • Guided Problem Solving

GPS Student Page 17, Exercise 31

A fitness club requires a $100 initiation fee and dues of $25 each month. Write an expression for the cost of membership for \( n \) months. Then find the cost of membership for one year.

Understand the Problem

1. What is the initiation fee for the club? 
2. What are the monthly dues for the club? 
3. What does the variable \( n \) represent? 
4. You are asked to write an expression. What does this expression represent? 
5. What are you asked to find? 

Make and Carry Out a Plan

6. What operation must you use to find the amount of dues for \( n \) months? 
7. Write an expression to represent the cost of dues for \( n \) months. 
8. The total cost of membership for \( n \) months includes the initiation fee and the cost of monthly dues for \( n \) months. What operation must you use to find the total cost of membership? 
9. Write an expression for the total cost of membership for \( n \) months, including the initiation fee. 
10. Evaluate the expression in Step 9 for \( n = 12 \), the number of months in a year. 
11. Simplify the expression to find the cost of membership for one year. 

Check the Answer

12. Look at the expression you found in Step 11. Which operation do you perform first? 

Solve Another Problem

13. Carly belongs to a book-of-the-month club. She paid $10 to sign up and then pays $5 for a new book each month. Write an expression for the cost of belonging to the club for \( n \) months. Then find the cost of belonging to the club for 8 months.
Practice 1-3

Evaluate each expression.

1. $xy$, for $x = 3$ and $y = 5$  
2. $24 - p \cdot 5$, for $p = 4$
3. $5a + b$, for $a = 6$ and $b = 3$  
4. $6x$, for $x = 3$
5. $9 - k$, for $k = 2$  
6. $63 \div p$, for $p = 7$
7. $2 + n$, for $n = 3$  
8. $3m$, for $m = 11$
9. $10 - r + 5$, for $r = 9$
10. $m + n \div 6$, for $m = 12$ and $n = 18$
11. $1221 \div x$, for $x = 37$  
12. $10 - x$, for $x = 3$
13. $4m + 3$, for $m = 5$  
14. $35 - 3x$, for $x = 10$
15. $851 - p$, for $p = 215$
16. $18a - 9b$, for $a = 12$ and $b = 15$
17. $3ab - c$, for $a = 4$, $b = 2$, and $c = 5$
18. $\frac{ab}{2} + 4c$, for $a = 6$, $b = 5$, and $c = 3$
19. $\frac{rst}{3}$, for $r = 9$, $s = 2$, and $t = 4$
20. $x(y + 5) - z$, for $x = 3$, $y = 2$, and $z = 7$
21. Elliot is 58 years old.
   a. Write an expression for the number of years by which Elliot’s age exceeds that of his daughter, who is $y$ years old.
   b. If his daughter is 25, how much older is Elliot?
22. A tree grows 5 in. each year.
   a. Write an expression for the tree’s height after $x$ years.
   b. When the tree is 36 years old, how tall will it be?
Reteaching 1-3

Writing and Evaluating Expressions

Evaluate \(a(b + 4) - c\), for \(a = 2, b = 5,\) and \(c = 12\).

\[a(b + 4) - c\]
\[= 2(5 + 4) - 12\]
\[= 2(9) - 12\]
\[= 18 - 12\]
\[= 6\]

Replace the variables.
Work within grouping symbols.
Multiply.
Subtract.

Evaluate each expression.

1. \(2n - 7,\) for \(n = 8\)

2. \(4ab,\) for \(a = 2\) and \(b = 5\)

3. \(\frac{x + y}{3},\) for \(x = 7\) and \(y = 8\)

4. \(2(m + n),\) for \(m = 3\) and \(n = 2\)

5. \(37 - 5h,\) for \(h = 7\)

6. \(\frac{6}{a},\) for \(a = 3\) and \(b = 7\)

7. \(4x + 5y - 3z,\) for \(x = 3, y = 4,\) and \(z = 2\)

8. \(15a - 2(b + c),\) for \(a = 2, b = 3,\) and \(c = 4\)

9. \(7p + q(3 + r),\) for \(p = 3, q = 2,\) and \(r = 1\)

10. \(\frac{36}{j} - 4(k + l),\) for \(j = 2, k = 1,\) and \(l = 3\)

11. \(x + 3y - 4(z - 3),\) for \(x = 4, y = 6,\) and \(z = 5\)

12. \((4 + d) - e(9 - f),\) for \(d = 7, e = 4, f = 8\)

13. \(3a - 2b + b(6 - 2),\) for \(a = 4, b = 2\)

14. \(r(p + 3) + q(p - 1),\) for \(p = 7, q = 4, r = 3\)
Enrichment 1-3

Equal Expressions

The value of a variable expression depends upon the value of the variable. By choosing the correct value, you can cause two expressions to be equal. The expressions $x - 6$ and $2x - 11$, for example, both have the same value when $x = 5$.

\[
\begin{align*}
x - 6 &= 5 - 6 \\
&= -1 \\
2x - 11 &= 2(5) - 11 \\
&= 10 - 11 \\
&= -1
\end{align*}
\]

Complete the tables for the given values of the variables. Then in the space to the right, name the value of the variable for which the two expressions are equal.

1. \[
\begin{array}{c|cccccccc}
& 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 \\
\hline
5 + k & & & & & & & & \\
2k - 9 & & & & & & & & \\
\end{array}
\]

$k = \underline{5}$

2. \[
\begin{array}{c|cccccccc}
& 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
10 - x & & & & & & & & \\
3x - 2 & & & & & & & & \\
\end{array}
\]

$x = \underline{3}$

3. \[
\begin{array}{c|cccccccc}
& 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
2n - 3 & & & & & & & & \\
27 - 3n & & & & & & & & \\
\end{array}
\]

$n = \underline{5}$

4. \[
\begin{array}{c|cccccccc}
& 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
5h + 7 & & & & & & & & \\
6h + 1 & & & & & & & & \\
\end{array}
\]

$h = \underline{0}$

5. \[
\begin{array}{c|cccccccc}
& 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
29 - 2a & & & & & & & & \\
3a + 4 & & & & & & & & \\
\end{array}
\]

$a = \underline{3}$

6. Explain why there is no value of $x$ for which the expressions $x + 3$ and $x + 4$ are equal.
Lesson 1-4

Integers and Absolute Value

Lesson Objectives

- Represent, graph, and order integers
- Find opposites and absolute values

NAEP 2005 Strand: Number Properties and Operations

Topic: Number Sense

Local Standards: ________________________________

Vocabulary

Opposites are ________________________________

Integers are ________________________________

An absolute value is ________________________________

Example

1. Representing Negative Numbers Write a number to represent the temperature shown by the thermometer.

The thermometer shows [ ] degrees Celsius below zero, or [ ].

Quick Check

1. Temperature Seawater freezes at about 28°F, or about 2 degrees Celsius below zero. Write a number to represent the Celsius temperature.
Examples

2 Graphing on a Number Line  Graph 2, −2, and −3 on a number line. Compare the numbers and order the numbers from least to greatest.

−2 is □ units to the left of 0.

−3 is □ units to the left of 0.  2 is □ units to the right of 0.

−3 is to the left of −2, and −2 is to the left of 2, so −3 < −2 < 2.

The numbers from least to greatest are □, □, □.

3 Finding Absolute Value  Use a number line to find |−5| and |5|.

□ units from 0  □ units from 0

|−5| = □  |5| = □

Quick Check

2. Graph 0, 2, and −6 on a number line. Compare the numbers and order them from least to greatest.

−6 < 0 < 2
The numbers from least to greatest are □, □, □.

3. Write |−10| in words. Then find |−10|.
Graph the set of numbers on a number line. Then order the numbers from least to greatest.

\(-2, 8, -9\)

**Understand the Problem**

1. What are you asked to do? ______________________________

2. What are the three numbers? ______________________________

**Make and Carry Out a Plan**

3. Draw a number line from \(-10\) to \(10\) in the space below.

4. Negative numbers are to the left of zero and positive numbers are to the right of zero. Plot \(-2, 8,\) and \(-9\) on the number line.

5. Numbers on a number line increase in value from left to right. Which number is farthest to the left on the number line? ______________________________

6. Order the numbers from least to greatest. ______________________________

**Check the Answer**

7. To check your answer, find the absolute value of each negative number.

\[|\text{negative number}|\]

The negative number with the greatest absolute value comes first when ordering numbers from least to greatest.

**Solve Another Problem**

Graph the set of numbers on a number line. Then order the numbers from least to greatest.

8. \(4, -3, -8\) ______________________________
Graph each set of numbers on a number line. Then order the numbers from least to greatest.

1. \(-4, -8, 5\)
2. \(3, -3, -2\)
3. \(0, -9, -5\)
4. \(-7, -1, -6\)

Write an integer to represent each quantity.

5. 5 degrees below zero
6. 2,000 ft above sea level
7. a loss of 12 yd
8. 7 strokes under par

Simplify each expression.

9. the opposite of \(-15\)
10. \(|-9|\)
11. \(|-25|\)
12. the opposite of \(|-8|\)
13. \(|-31|\)
14. \(|847|\)

Write the integer represented by each point on the number line.

15. \(A\)
16. \(B\)
17. \(C\)
18. \(D\)
19. \(E\)

Compare. Use >, <, or = to complete each statement.

20. \(-3\) \(\quad\) 4
21. \(5\) \(\quad\) 1
22. \(-2\) \(\quad\) -6
23. 7 \(\quad\) \(|8|\)
24. \(|-2|\) \(\quad\) \(|2|\)
25. \(|-1|\) \(\quad\) -6
26. \(|4|\) \(\quad\) \(|-5|\)
27. 0 \(\quad\) \(|-7|\)
Reteaching 1-4  
Integers and Absolute Value

Compare. Use >, <, or = to complete each statement.

a. \(-4 \quad \square \quad -2\)
   
   Graph \(-4\) and \(-2\) on the number line.

   \[\begin{array}{cccccccc}
   -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
   \end{array}\]

   A number on the left is less than a number on the right.
   Thus, \(-4\) is less than \(-2\).
   \(-4 < -2\)

b. \(|-4| \quad \square \quad |-2|\)
   
   The absolute value of a number is its distance from zero on the number line.

   \[\begin{array}{cccccccc}
   -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
   \end{array}\]

   Thus \(|-4| = 4\) and \(|-2| = 2\).
   Since \(4 > 2\), \(|-4| > |-2|\)

Compare. Use >, <, or = to complete each statement.

1. \(-3 \quad \square \quad -2\)
2. \(-5 \quad \square \quad 1\)
3. \(0 \quad \square \quad -2\)
4. \(1 \quad \square \quad 0\)
5. \(1 \quad \square \quad -1\)
6. \(-5 \quad \square \quad -3\)
7. \(|-3| \quad \square \quad 0\)
8. \(|-2| \quad \square \quad |-5|\)
9. \(|-3| \quad \square \quad 2\)
10. \(|-6| \quad \square \quad 6\)
11. \(|3| \quad \square \quad |-2|\)
12. \(|-7| \quad \square \quad 0\)
13. \(-3 \quad \square \quad |-3|\)
14. \(4 \quad \square \quad |-2|\)
15. \(|-2| \quad \square \quad 3\)
16. \(|-5| \quad \square \quad 3\)
17. \(|8| \quad \square \quad |-8|\)
18. \(-6 \quad \square \quad -4\)
19. \(5 \quad \square \quad |-4|\)
20. \(-3 \quad \square \quad -5\)
21. \(|2| \quad \square \quad |-3|\)
22. \(|-1| \quad \square \quad |1|\)
23. \(|-3| \quad \square \quad |-1|\)
24. \(-1 \quad \square \quad 2\)
Enrichment 1-4

Absolute Value Equations

The absolute value of an integer is its distance from zero on a number line. You can use that fact to solve absolute value equations.

Example 1  Solve \( |n| = 2 \).

Solution  Both \(-2\) and \(2\) are at a distance of 2 units from 0 on the number line. Therefore, \( n = -2 \) or \( n = 2 \).

Example 2  Solve \( |n - 2| = 3 \).

Solution  Both \(-1\) and \(5\) are at a distance of 3 units from 2 on the number line. Therefore, \( n = -1 \) or \( n = 5 \).

Solve by naming the possible values of \( n \). Use the number line.

1. \( |n| = 4 \)  
   \[ n = _____ \text{ or } _____ \]
2. \( |n| = 1 \)  
   \[ n = _____ \text{ or } _____ \]
3. \( |n| = 8 \)  
   \[ n = _____ \text{ or } _____ \]
4. \( |n - 3| = 1 \)  
   \[ n = _____ \text{ or } _____ \]
5. \( |n - 2| = 5 \)  
   \[ n = _____ \text{ or } _____ \]
6. \( |n - (-4)| = 6 \)  
   \[ n = _____ \text{ or } _____ \]
7. \( |5 - n| = 1 \)  
   \[ n = _____ \text{ or } _____ \]
8. \( |2 - n| = 3 \)  
   \[ n = _____ \text{ or } _____ \]
9. \( |-6 - n| = 2 \)  
   \[ n = _____ \text{ or } _____ \]
10. \( |n| = 0 \)  
    \[ n = _____ \]
11. \( |n - 5| = 0 \)  
    \[ n = _____ \]
12. \( |-3 - n| = 0 \)  
    \[ n = _____ \]
13. \( |-n| = 10 \)  
    \[ n = _____ \]
14. \( |n - (-6)| = 3 \)  
    \[ n = _____ \text{ or } _____ \]
15. \( |n - 7| = 3 \)  
    \[ n = _____ \text{ or } _____ \]
16. \( |-4 - n| = 2 \)  
    \[ n = _____ \text{ or } _____ \]
17. \( |n - 2| = 4 \)  
    \[ n = _____ \text{ or } _____ \]
18. \( |1 - n| = 0 \)  
    \[ n = _____ \]
19. How do you know that the equation \( |n| = -3 \) has no solution?
1B: Reading Comprehension

Study Skill When you read a paragraph in mathematics, read it twice. Read it the first time to get an overview of the content. Read it a second time to find the essential details and information. Write down key words that tell you the topic for each paragraph.

Read the passage below and answer the questions that follow.

Algebra is a part of mathematics that uses variables as well as the operations that combine variables. Some operations in algebra are the ones you learned in arithmetic (+, −, ×, ÷). In algebra, however, you might add two variables such as \( a \) and \( b \). Then you could substitute different values for these variables. So, for example, \( a + b \) can represent 3 + 4 or 13.5 + 24.7 or any other numbers you choose.

Two different people have commonly been called “the father of algebra.” One is Diophantus, a Greek mathematician who lived in the third century. He was the first to use symbols to represent frequently used words. The other is the Arab mathematician Al-Khowarizmi. In the ninth century, he published a clear and complete explanation of how to solve an equation. Our word “algebra” comes from the word, \( al-jabr \), which appears in the title of his work.

1. What is the subject of the first paragraph? What is the subject of the second paragraph?

2. How are numbers used in the passage?

3. What are the names of the mathematicians mentioned in the passage?

4. What title do they share?

5. How much time passed between the lives of these two mathematicians?

6. According to this passage, what is the same in arithmetic and algebra?

7. Which operations are named in this passage?

8. What was the origin of the word \( algebra \)?

9. **High-Use Academic Words** In the first paragraph of the passage, what does it mean to substitute?
   - a. to use in place of another
   - b. to prove
Lesson 1-5

Adding Integers

**Key Concepts**

**Addition of Opposites**
The sum of an integer and its opposite is 0.

**Arithmetic**

\[ 1 + (-1) = \]

\[ -1 + 1 = \]

**Algebra**

\[ x + (-x) = \]

\[ -x + x = \]

**Adding Integers**

**Same Sign** The sum of two positive integers is . The sum of two negative integers is .

**Different Signs** To add two integers with different signs, find the difference of their absolute values. The sum has the sign of the integer with the absolute value.

**Example**

1. **Using Tiles to Add Integers** Use tiles to find \((-7) + 3\).

   Model the sum.

   Group and remove zero pairs.

   There are negative tiles left.

   \((-7) + 3 = \)

**Quick Check**

1. Use tiles to find each sum.

   a. \(-1 + 4\)

   Model the sum.

   Group and remove zero pairs.

   \[\text{There are negative tiles left.}\]

   \[\text{\(-1 + 4 = \)}\]

   b. \(7 + (-3)\)

   Model the sum.

   Group and remove zero pairs.

   \[\text{There are negative tiles left.}\]

   \[\text{\(7 + (-3) = \)}\]

   c. \(-2 + (-2)\)

   Model the sum.

   Group and remove zero pairs.

   \[\text{There are negative tiles left.}\]

   \[\text{\(-2 + (-2) = \)}\]
Examples

2 Using a Number Line From the surface, a diver goes down 20 feet and then comes back up 4 feet. Find \(-20 + 4\) to find where the diver is.

Start at 0. To represent \(-20\), move left \(\boxed{20}\) units. To add positive 4, move right \(\boxed{4}\) units to \(\boxed{0}\).

\(-20 + 4 = \boxed{-16}\)

The diver is \(\boxed{-16}\) feet below the surface.

3 Using the Order of Operations Find \(-7 + (-4) + 13 + (-5)\).

\(-7 + (-4) + 13 + (-5)\) Add from left to right.

\(\boxed{-7} + 13 + (-5)\) The sum of the two negative integers is \(\boxed{11}\).

\(\boxed{11} + (-5)\) Since \(\boxed{-5}\) has the greater absolute value, the sum is \(\boxed{6}\).

\(\boxed{6} - 4\) Since \(\boxed{6}\) has the greater absolute value, the sum is \(\boxed{2}\).

\(-7 + (-4) + 13 + (-5) = \boxed{2}\).

Quick Check

2. Use this number line to find each sum.

\[\begin{array}{ccc}
-10 & -8 & -6 & -4 & -2 & 0 & 2 & 4 & 6 & 8 & 10 \\
\end{array}\]

a. \(2 + (-6)\)

\(\boxed{-4}\)

b. \(-4 + 9\)

\(\boxed{5}\)

c. \(-5 + (-1)\)

\(\boxed{-6}\)

3. a. \(1 + (-3) + 2 + (-10)\)

\(\boxed{-10}\)

b. \(-250 + 200 + (-100) + 220\)

\(\boxed{-30}\)

4. Geography An earthquake monitor in Hockley, Texas, is located in a salt mine at an elevation of \(-416\) m. The elevation of an earthquake monitor in Piñon Flat, California, is \(1,696\) m higher than the monitor in Hockley. Find the elevation of the monitor in Piñon Flat.

\(\boxed{-252}\)
GPS 1-5 • Guided Problem Solving

Finance Maria had $123. She spent $35, loaned $20 to a friend, and received her $90 paycheck. How much does she have now?

Understand the Problem
1. How much money did Maria start with? __________________________
2. What amount did she spend? __________________________
3. How much money did she loan to a friend? __________________________
4. What was the amount of Maria’s paycheck? __________________________

Make and Carry Out a Plan
5. Look at the amounts below. Tell whether each amount should be added to or subtracted from Maria’s $123.
   a. $35 __________________________
   b. $20 __________________________
   c. $90 __________________________
6. Write an expression to show the original amount of $123 and the amounts that should be added or subtracted. __________________________
7. Simplify the expression to find the amount of money Maria has now. __________________________

Check the Answer
8. To check your work, start with the amount you found in Step 7 and work backward. Subtract 90, add 20, and add 35. Is your result the same as the amount Maria started with?

________________________

Solve Another Problem
9. Alec had $55. He earned $25 mowing lawns in his neighborhood. He spent $10 on a new baseball card for his collection. Then he spent $6 on lunch with a friend. How much money does Alec have now? __________________________
Write a numerical expression for each of the following. Then find the sum.

1. climb up 26 steps, then climb down 9 steps

2. earn $100, spend $62, earn $35, spend $72

Find each sum.

3. \(-8 + (-3)\)
4. \(6 + (-6)\)
5. \(-12 + (-17)\)
6. \(9 + (-11)\)
7. \(-4 + (-6)\)
8. \(18 + (-17)\)
9. \(-8 + 8 + (-11)\)
10. \(12 + (-7) + 3 + (-8)\)
11. \(-15 + 7 + 15\)
12. \(0 + (-11)\)
13. \(6 + (-5) + (-4)\)
14. \(-5 + (-16) + 5 + 8 + 16\)

Without adding, tell whether each sum is positive, negative, or zero.

15. \(192 + (-129)\)
16. \(-417 + (-296)\)
17. \(-175 + 87\)

Evaluate each expression for \(n = -12\).

18. \(n + 8\)
19. \(n + (-5)\)
20. \(12 + n\)

Compare. Write \(>\), \(<\), or \(=\) to complete each statement.

21. \(-7 + 5 \quad \square \quad 3 + (-6)\)
22. \(4 + (-9) \quad \square \quad 6 + (-7) + (-4)\)
23. An elevator went up 15 floors, down 9 floors, up 11 floors, and down 19 floors. Find the net change. 
24. The price of a share of stock started the day at $37. During the day it went down $3, up $1, down $7, and up $4. What was the price of a share at the end of the day?
Use tiles and the rules for adding integers to find each sum.

a. \(-4 + (-3)\)

\[
\begin{array}{ccc}
\text{negative} & \text{negative} & \text{positive} \\
\text{negative} & \text{negative} & \\
\end{array}
\]

Four negative tiles plus 3 negative tiles gives 7 negative tiles.
\(-4 + (-3) = -7\)

The sum of two negative integers is negative.

b. \(-8 + 3\)

\[
\begin{array}{ccc}
\text{negative} & \text{negative} & \text{negative} & \text{negative} & \text{positive} \\
\text{negative} & \text{negative} & \text{negative} & \text{negative} & \text{negative} & \text{negative} & \text{negative} & \text{positive} \\
\end{array}
\]

Remove zero pairs

Since the signs of the integers are different, you must remove zero pairs. The number of tiles left is the number of negative tiles \(|-8|\) minus the number of positive tiles \(|3|\). Thus, you can always subtract the absolute values of the numbers to find how many tiles will be left.

\(|-8| - |3| = 5\)

Since there are more negative tiles than positive tiles, \(|-8| > |3|\), there are negative tiles left after you subtract zero pairs. Thus, the sum is negative.

\(-8 + 3 = -5\)

---

Use rules or tiles to find each sum.

1. \(9 + (-12)\)  
2. \(-4 + 10\)  
3. \(-1 + (-8)\)

4. \((-6) + (-11)\)  
5. \((-5) + 15\)  
6. \(2 + (-14)\)

7. \((-3) - 6\)  
8. \(-(-2) + 9\)  
9. \((-2) - 4\)

10. \(-5 - (-4)\)  
11. \(7 + (-2)\)  
12. \(16 + (-6)\)
Enrichment 1-5

Clock Numbers

A finite number system is one that contains a limited number of numbers. The finite number system on a clock face consists of the numbers from 1 to 12. The clock system is called Modulo-twelve, which is abbreviated \( \text{mod} \ 12 \).

Since “14” o’clock equals 2 o’clock, we can write
\[ 14 = 2(\text{mod} \ 12) \]

The integer 14 is equivalent to the number 2 in the mod 12 system. Every integer has an equivalent in mod 12. To find the equivalent of an integer, add or subtract a multiple of 12 to obtain a number between 1 and 12.

Examples
\[ 55 = 55 - 4(12) = 55 - 48 = 7(\text{mod} \ 12) \]
\[ -13 = -13 + 2(12) = -13 + 24 = 11(\text{mod} \ 12) \]

Find the mod 12 equivalent. Each answer must be a number from 1 to 12.

1. 18
2. 85
3. -5
4. -64
5. 149
6. -97

The numbers 13, 25, and 37 are all equivalent to 1 \( \text{mod} \ 12 \). When integers have the same equivalent, they are said to be congruent. The numbers 13, 25, and 37 are congruent in mod 12.

Write four integers, two positive and two negative, that are congruent in mod 12 to the given number.

7. 3
8. 8
9. 12

To add in mod 12, find the sum in the usual manner. Then write the mod 12 equivalent.

Find the sum in mod 12.

10. 6 + 11
11. 9 + 5 + 12
12. 4 + (-7) + (-8)
13. 3 + 11 + (-5) + (-16)
14. -35 + (-47) + 28 + (-77)
15. 29 + (-33) + (-2) + 14
16. -22 + (-11) + (-5) + (-19)
Lesson 1-6

Subtracting Integers

Lesson Objectives
✓ Use models to subtract integers
✓ Use a rule to subtract integers

NAEP 2005 Strand: Number Properties and Operations
Topic: Number Operations
Local Standards: ____________________________

Key Concepts

Subtracting Integers
To subtract an integer, add its __________.

Arithmetic
\[2 - 5 = 2 + (\square) = -3\]
\[2 - (\square) = 2 + 5 = 7\]

Algebra
\[a - b = a + (\square)\]
\[a - (\square) = a + b\]

Examples

1. Using Tiles to Subtract Integers  Find \(-7 - (-5)\).
   
   ![Using Tiles to Subtract Integers](image)
   
   Number of tiles subtracted: 5
   Number of tiles remaining: 2
   
   \(-7 - (-5) = \square\)

2. Using Zero Pairs to Subtract Integers  Find \(2 - 8\).
   
   ![Using Zero Pairs to Subtract Integers](image)
   
   Number of tiles subtracted: 8
   Number of tiles remaining: 10
   
   \(2 - 8 = \square\)
Using a Rule to Subtract Integers  An airplane left Houston, Texas, where the temperature was 42°F. When the airplane landed in Anchorage, Alaska, the temperature was 50°F lower. What was the temperature in Anchorage?

\[ 42 - 50 \]

Write an expression.

\[ 42 - 50 = 42 + (\text{ }) \]

To subtract 50, add its \( \text{ } \) .

Simplify.

\[ = \text{ } \]

The temperature in Anchorage was \( \text{ } \).

Quick Check

1. Use tiles to find each difference.
   a. \(-7 - (-2)\)  
   ![Tile representation of \(-7 - (-2)\)]
   \(-7 - (-2) = \text{ } \)
   
   b. \(-4 - (-3)\)  
   ![Tile representation of \(-4 - (-3)\)]
   \(-4 - (-3) = \text{ } \)
   
   c. \(-8 - (-5)\)  
   ![Tile representation of \(-8 - (-5)\)]
   \(-8 - (-5) = \text{ } \)

2. Use tiles to find each difference.
   a. \(4 - 8\)  
   ![Tile representation of \(4 - 8\)]
   \(4 - 8 = \text{ } \)
   
   b. \(-1 - 5\)  
   ![Tile representation of \(-1 - 5\)]
   \(-1 - 5 = \text{ } \)
   
   c. \(-2 - (-7)\)  
   ![Tile representation of \(-2 - (-7)\)]
   \(-2 - (-7) = \text{ } \)

3. Find each difference.
   a. \(32 - (-3)\)  
   
   b. \(-40 - 66\)  
   
   c. \(2 - 48\)  
   
   d. **Weather** The lowest temperature ever recorded on the moon was about \(-170°C\). The lowest temperature ever recorded in Antarctica was \(-89°C\). Find the difference in the temperatures.
Scores Suppose you have a score of 35 in a game. You get a 50-point penalty. What is your new score?

Understand the Problem

1. What is your original score? __________________________
2. How many points is your penalty? ______________________
3. What are you asked to find? ____________________________

Make and Carry Out a Plan

4. Is your original score positive or negative? _______________
5. Should you add or subtract your penalty from your original score?
   __________________________
6. Write an expression to show how to combine a 50-point penalty with the original 35-point score.
   __________________________
7. Simplify the expression from Step 6 to find your new score.
   __________________________

Check the Answer

8. Write the expression from Step 6 as a sum.
   __________________________
9. Find the sum. __________________________

Solve Another Problem

10. Trevor has a score of 55 in a game.
    He gets a 75-point penalty. What is his new score? ______________

---

Pre-Algebra Lesson 1-6
Guided Problem Solving
Use rules to find each difference.

1. $8 - 12$
2. $13 - 6$
3. $9 - (-12)$

4. $57 - 39$
5. $-173 - 162$
6. $71 - (123)$

7. $51 - 89$
8. $-222 - (-117)$
9. $843 - 677$

10. $-98 - 183$
11. $366 - (-429)$
12. $-83 - (-48) - 65$

Find each difference.

13. $6 - 9$
14. $14 - 8$
15. $-15 - 3$

16. $-25 - 25$
17. $-16 - (-16)$
18. $32 - (-17) - 32$

Round each number. Then estimate each sum or difference.

19. $-57 + (-98)$
20. $448 - 52$
21. $-191 + (-511)$

22. $-361 - (-58)$
23. $888 + 1,177$
24. $-484 - 1,695$

Write a numerical expression for each phrase. Then simplify.

25. A balloon goes up 2,300 ft, then goes down 600 ft.

26. You lose $50, then spend $35.

27. The Glasers had $317 in their checking account. They wrote checks for $74, $132, and $48. What is their checking account balance?
Reteaching 1-6

Subtracting Integers

a. Find \(-7 - (-3)\) and \(-7 + 3\). Compare.

\[
\begin{align*}
-7 - (-3) & \quad -7 + 3 \\
\text{Start with 7 negative tiles and take away 3 negative tiles.} & \quad \text{Add three positive tiles. Remove zero pairs.}
\end{align*}
\]

With both you start with 7 negative tiles. Taking away 3 negative tiles has the same effect as adding 3 positive tiles and removing zero pairs.

\[-7 - (-3) = -7 + 3 = -4\]

b. Find \(-4 - 2\) and \(-4 + (-2)\). Compare.

\[
\begin{align*}
-4 - 2 & \quad -4 + (-2) \\
\text{With both you start with 4 negative tiles. Adding two zero pairs and taking away two positive tiles has the same effect as adding two negative tiles.} & \\
-4 - 2 = -4 + (-2) = -6
\end{align*}
\]

Use rules for subtracting integers to find each difference. Use tiles to help.

1. \(-5 - (-3) = -5 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}\)

2. \(-8 - 6 = -8 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}\)

3. \(3 - (-9) = 3 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}\)

4. \(-2 - (-7) = -2 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}\)

5. \(4 - 10 = 4 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}\)

6. \(1 - (-6) = 1 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}\)

7. \(-9 - 5 = -9 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}\)

8. \(-6 - (-2) = -6 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}\)

9. \(7 - 8 = 7 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}\)
Enrichment 1-6

Time Lines

A time line is a number line marked off in dates rather than in integers. On the History of Mathematics time line below, dates labeled B.C. fall where the negative integers normally lie. Dates labeled A.D. replace the positive integers. Years given are dates of birth.

\[
\begin{array}{c|c|c|c|c|c|c|c|c|}
& 569 & 429 & 330 & 287 & 0 & 98 & 250 & 526 & 640 \\
B.C. & Plato & Euclid & & & & Hero & Ptolemy & & \\
A.D. & & & & & & & & & Brahmagupta \\
\end{array}
\]

Find the number of years between the given events. Write a subtraction expression. Then simplify.

1. the births of Euclid and Hero
   \[569 - 287 = 282\]

2. the births of Pythagoras and Archimedes
   \[287 - 429 = -142\]

3. the births of Brahmagupta and Ptolemy
   \[287 - 330 = -43\]

4. Legend has it that Rome was founded in 753 B.C. How many years after the founding of Rome was Plato born?
   \[287 - 753 = -466\]

5. One mathematician was born as many years before Ptolemy as Aryabhata was born after Ptolemy. Which one?
   \[526 - 287 = 239\]

6. Which mathematician was born 1,069 years before Brahmagupta?
   \[287 - (287 - 1069) = 1069\]

Use the number line below to construct a time line. Write the letters of the given events below the appropriate tic mark. Above the line, write dates. Then, choose five other events relating to you, your family or friends, and include these on the time line. Also, include the year of your birth.

7. 
   \[
   \begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|}
   & & & & & & & & & & & \\
   \end{array}
   \]
Lesson 1-7

Lesson Objectives
1. Write rules for patterns
2. Make predictions and test conjectures

NAEP 2005 Strand: Algebra
Topic: Patterns, Relations, and Functions
Local Standards: ________________________________

Vocabulary
Inductive reasoning is __________________________________________________________________________

A conjecture is ____________________________________________________________________________

A counterexample is __________________________________________________________________________

Examples
1. Reasoning Inductively Use inductive reasoning. Make a conjecture about the next figure in the pattern. Then draw the figure.

Observation: The circles are rotating ________ within the square.

Conjecture: The next figure will have a shaded circle at the ________

2. Writing Rules for Patterns Write a rule for each number pattern.
   a. 0, –4, –8, –12,… Start with 0 and ________ repeatedly.
   b. 4, –4, 4, –4,… Alternate ________ and its ________.
   c. 1, 2, 4, 8, 10,… Start with ________. Alternate ________ and ________.
Extending a Pattern  Write a rule for the number pattern 110, 100, 90, 80,....
Find the next two numbers in the pattern.

110, 100, 90, 80,  

The first number is 110.

The next numbers are found by subtracting 10.

The rule is \[ \text{Start with } \underline{110} \text{ and } \underline{10} = \underline{100}, \text{ and } \underline{10} = \underline{100}. \]

Analyzing Conjectures  Is the conjecture correct or incorrect? If it is incorrect, give a counterexample.

Every triangle has three sides of equal length.

The conjecture is \[ \underline{\text{Every triangle has three sides of equal length.}} \]

Quick Check

1. Make a conjecture about the next figure in the pattern at the right.
   Then draw the figure.
   | △ | □ | □ |
   |   |   |   |

2. Write a rule for each pattern.
   a. 4, 9, 14, 19, ...
   b. 3, 9, 27, 81, ...
   c. 1, 1, 2, 3, 5, 8, ...

3. Write a rule for the pattern 1, 3, 5, 7, .... Find the next two numbers in the pattern.

4. Is each conjecture correct or incorrect? If it is incorrect, give a counterexample.
   a. The last digit of the product of 5 and a whole number is 0 or 5.
   b. A number and its absolute value are always opposites.
   c. The next figure in the pattern has 25 dots.
Reasoning Is the conjecture correct or incorrect? If incorrect, give a counterexample.

A whole number is divisible by 3 if the sum of its digits is divisible by 3.

Understand the Problem

1. Write the conjecture in your own words.

Make and Carry Out a Plan

2. An example of a whole number whose digits have a sum that is divisible by 3 is 27. List three other such whole numbers.

3. Show that each whole number you found in Step 2 is divisible by 3.

4. The sum of the digits of 102 is divisible by 3.
   Also, 102 is divisible by 3. What are two other three-digit numbers for which the sum of the digits is divisible by 3?

5. Is each number you named in Step 4 also divisible by 3?

6. Based on your trials, does the conjecture seem correct or incorrect? Explain.

Check the Answer

7. Test the conjecture using the number 5,112.
   Is the sum of the digits divisible by 3? Is 5,112 divisible by 3?

Solve Another Problem

8. Is the conjecture correct or incorrect? If it is incorrect, give a counterexample.
   A whole number is divisible by 2 if the sum of its digits is divisible by 2.
Write a rule for each pattern. Find the next three numbers in each pattern.

1. 3, 6, 9, 12, 15, _____, _____, _____
   Rule: __________________________

2. 1, 2, 4, 8, 16, _____, _____, _____
   Rule: __________________________

3. 6, 7, 14, 15, 30, 31, _____, _____, _____
   Rule: __________________________

4. 34, 27, 20, 13, 6, _____, _____, _____
   Rule: __________________________

Is each statement correct or incorrect? If it is incorrect, give a counterexample.

5. All roses are red.

6. A number is divisible by 4 if its last two digits are divisible by 4.

7. The difference of two numbers is always less than at least one of the numbers.

Describe the next figure in each pattern. Then draw the figure.

8. [Figure 1]

9. [Figure 2]

10. [Figure 3]

11. [Figure 4]
Reteaching 1-7

Inductive Reasoning

The sum of two numbers is always at least as great as either number. Is the statement correct or incorrect? If incorrect, give a counterexample.

Try some examples.

\[ 2 + 8 = 10 \quad 10 \geq 8 \text{ and } 10 \geq 2 \]
\[ 365 + 241 = 606 \quad 606 \geq 365 \text{ and } 606 \geq 241 \]

The conjecture seems correct. Try different kinds of numbers. Although the numbers in the second trial are much larger than those in the first, all are whole numbers. Try zero, fractions, and negative numbers.

\[ \frac{3}{8} + \frac{1}{8} = \frac{1}{2} \quad \frac{1}{2} \geq \frac{3}{8} \text{ and } \frac{1}{2} \geq \frac{1}{8} \]
\[ -4 + 7 = 3 \quad 3 \geq -4 \text{ but } 3 \text{ is not at least as great as } 7 \]

The conjecture is incorrect and \(-4 + 7 = 3\) is a counterexample.

Is each conjecture correct or incorrect? If incorrect, give a counterexample.

1. The difference of two numbers is less than or equal to each number.

2. The sum of two negative numbers is always less than each number.

3. The sum of 5 and any positive integer is divisible by 5.

4. A number is divisible by 10 if its last digit is 0.

5. The sum of a number and its absolute value is always 0.

6. The next number in the pattern 2, 4, 8, \ldots \text{ is } 10.

7. Every even number is divisible by 4.

8. The next number in the pattern 5, 3, 1, \ldots \text{ is } -1.
Enrichment 1-7

Bode’s Pattern

Occasionally, a scientist discovers a pattern of numbers that seems to suggest a natural law. The scientist must prove that the pattern is not simply accidental but that there is a reason behind it.

The table lists the planets known in 1772 and their relative distances from the sun, taking the Earth’s distance as 10. In that year, the astronomer Johann Bode discovered an amazing pattern of numbers that closely matched the planetary distances.

1. To find Bode’s pattern, start with 1.5 and double each term.
   1.5, 3, _____, _____, _____, _____

2. Now add 4 to each term.
   5.5, _____, _____, _____, _____, _____

3. With one exception, note the close correlation between the pattern and the relative distances in the table.
   a. Two planetary distances are off slightly. Which two?

   ____________________________

   b. What is the exception?

   ____________________________

4. In 1781, Uranus was discovered at a relative distance of 196 from the Sun. Calculate the next number in Bode’s pattern in Exercise 2. _____

   Does the pattern correctly predict the discovery of Uranus? _____

5. In 1801, Ceres, the first and largest of the asteroids or “minor” planets, was discovered at a relative distance of 28.

   Does the pattern correctly predict the discovery of Ceres? _____

6. In 1846, the planet Neptune was discovered at a relative distance of 301. Had Bode discovered a law of planetary distance? Explain.

   ____________________________
1E: Vocabulary Check

Study Skill  Strengthen your vocabulary. Use these pages and add cues and summaries by applying the Cornell Notetaking style.

Write the definition for each word at the right. To check your work, fold the paper back along the dotted line to see the correct answers.

Integers

Absolute value

Inductive reasoning

Conjecture

Counterexample
Write the vocabulary word for each definition. To check your work, fold the paper forward along the dotted line to see the correct answers.

The whole numbers and their opposites.

The distance of a number from zero on a number line.

Making conclusions based on patterns you observe.

A conclusion reached through inductive reasoning.

An example that proves a statement false.
Lesson 1-8

Look for a Pattern

Lesson Objective

- Find number patterns

NAEP 2005 Strand: Algebra

Topic: Patterns, Relations, and Function

Local Standards: ________________________________

Example

Each student on a committee of five students shakes hands with every other committee member. How many handshakes will there be in all?

Understand the Problem: How many hands does each committee member shake?

Make and Carry Out a Plan: Make a table to organize the numbers. Then look for a pattern.

The pattern is to add the number of new handshakes to the number of handshakes already made.

\[ \text{the number of handshakes by 1 student} \]
\[ 4 + \square = \square \text{ the number of handshakes by 2 students} \]

Make a table to extend the pattern to 5 students.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of original handshakes</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total number of handshakes</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There will be \[ \square \] handshakes in all.

Check the Answer: One way to check a solution is to solve the problem by another method. You can use a diagram to show the pattern visually.

There are \[ \square \] diagonals in the pentagon, so there will be \[ \square \] handshakes in all.
**Quick Check**

1. Suppose that the committee is made up of six people. How many handshakes would there be?

2. **a. Information** News spreads quickly at Riverdell High. Each student who hears a story repeats it 15 minutes later to two students who have not heard it yet, and then tells no one else. Suppose one student hears some news at 8:00 A.M. How many students will know the news at 9:00 A.M.?

   b. Suppose each student who hears the story repeats it in 10 minutes. How many students will know the news at 9:00 A.M.?
1-8 • Guided Problem Solving

Solve by looking for a pattern.

Students are to march in a parade. There will be one first grader, two second graders, three third graders, and so on, through the twelfth grade. How many students will march in the parade?

Understand the Problem

1. How many first graders will march in the parade?
2. How many second graders will march in the parade? Third graders?
3. What are you asked to find?

Make and Carry Out a Plan

4. Make a table to organize the information. Complete the table with the information you know about the number of first, second, and third graders.

<table>
<thead>
<tr>
<th>Grade</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students in the Parade</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

5. Look for a pattern in the number of students. What pattern do you see?

6. Use the pattern to complete the table above.

7. Add the number of students from each grade who will march in the parade. How many students will march in the parade?

Check the Answer

8. To check your answer, draw a diagram on a separate piece of paper. Use a dot to represent each student. In the first row of your diagram, draw the number of first graders, in the second row the number of second graders, and so on. The number of dots should be equal to the number of students from Question 7.

Solve Another Problem

9. At Highland Elementary School, one first grader, three second graders, five third graders and so on through the sixth grade are crossing guards. How many students are crossing guards?
Solve by looking for a pattern.

1. Each row in a window display of floppy disk cartons contains two more boxes than the row above. The first row has one box.
   a. Complete the table.

<table>
<thead>
<tr>
<th>Row Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boxes in the Row</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Boxes in the Display</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Describe the pattern in the numbers you wrote.

   ____________________________________________________________________________

   c. Find the number of rows in a display containing the given number of boxes.

   81  ________  144  ________  400  ________

   d. Describe how you can use the number of boxes in the display to calculate the number of rows.

   ____________________________________________________________________________

2. A computer multiplied 100 nines. You can use patterns to find the ones digit of the product.
   \[ 9 \times 9 \times 9 \times 9 \times \cdots \times 9 \]
   100 times

   a. Find the ones digit for the product of:

   1 nine  _____  2 nines  _____  3 nines  _____  4 nines  _____

   b. Describe the pattern.

   ____________________________________________________________________________

   ____________________________________________________________________________

   ____________________________________________________________________________

   ____________________________________________________________________________

   c. What is the ones digit of the computer’s product?  ________

3. Use the method of Exercise 2 to find the ones digit of the product when 4 is multiplied by itself 100 times.  ________
**Reteaching 1-8**

Margarita learned to dig clams over her vacation and got steadily better at finding clams each day. On the first day she found 2 clams, on the second day 5 clams, and on the third day 8. If she continued to improve at the same rate, how many clams did she find on the sixth day?

Make a table to organize the numbers. Then look for a pattern.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clams</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>More Than Day Before</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Margarita found 17 clams on the sixth day.

Phillipe got steadily better at playing ping pong on his vacation. The table shows the number of games he won the first three days. If he continued to improve at the same rate, how many games would he win on the sixth day?

1. Complete the table.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Games Won</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>More Than Day Before</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Solve the problem.

Jennifer improved her bike riding distance steadily while preparing for a race. The table shows the distance in miles she rode during the first three weeks of training. If she continues to improve at the same rate, how many miles will she be able to ride in the sixth week? How many more miles did she ride in week 6 than she rode in week 5?

3. Complete the table.

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles Traveled</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>More Than Week Before</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Solve the problems.
Two of the most important factors influencing temperature are elevation and latitude. (Latitude is position on the earth’s surface measured in degrees north or south of the equator, from 0° to 90°.)

<table>
<thead>
<tr>
<th>Elevation Rule</th>
<th>Latitude Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>For every 300-ft gain in elevation, subtract 1°F.</td>
<td>For every 2 degrees of latitude north or south of the equator, subtract 3°F.</td>
</tr>
</tbody>
</table>

For Exercises 1–8, refer to the table.

1. On average, how much warmer is Albuquerque than Portland due to latitude? ________________
2. On average, how much colder is Albuquerque than Portland due to altitude? ________________
3. Find the net difference. ________________
4. Which city is colder? By how much? ________________
5. How much warmer is Mount Massive than Chicago due to latitude? ________________
6. How much colder is Mount Massive than Chicago due to altitude? ________________
7. Find the net difference. ________________
8. Which location is colder? By how much? ________________
9. Moscow, Russia, has a latitude of 56°N and an altitude of 400 ft. Mexico City, Mexico, has a latitude of 20°N and an altitude of 7,300 ft. Which location is colder? By how much? ________________
10. Peking, China, has a latitude of 40°N and an altitude of 150 ft. St. Louis, Missouri, has a latitude of 38°N and an altitude of 450 ft. Which location is colder? By how much? ________________
Lesson 1-9

Multiplying and Dividing Integers

Lesson Objectives

- Multiply integers using repeated addition, patterns, and rules
- Divide integers using rules

NAEP 2005 Strand: Number Properties and Operations
Topic: Number Operations
Local Standards: ________________________________

Key Concepts

**Multiplying Integers**
The product of two integers with the same sign is _______.
The product of two integers with different signs is _______.
The product of zero and any integer is _______.

Examples

\[3(4) = \] \[3(-4) = \]
\[-3(-4) = \] \[-3(4) = \]
\[3(0) = \] \[-4(0) = \]

**Dividing Integers**
The quotient of two integers with the same sign is _______.
The quotient of two integers with different signs is _______.
Remember that division by zero is _______.

Examples

\[12 \div 3 = \] \[12 \div (-3) = \]
\[-12 \div (-3) = \] \[-12 \div 3 = \]

**Examples**

1. **Using Patterns to Multiply Integers**  Use a pattern to find each product.

   a. \[-2(7)\]
   
   \[2(7) = \] \[\text{Start with products you know.} \rightarrow 2(-7) = \]
   
   \[1(7) = \]
   
   \[0(7) = \]
   
   \[-1(7) = \] \[\text{Continue the pattern.} \rightarrow -1(-7) = \]
   
   \[-2(7) = \]
Using Rules to Multiply Integers  Multiply \((6)(-2)(-3)\).

\[6(-2)(-3) = \left(\phantom{-}\right)(-3)\]

Multiply from left to right. The product of a positive integer and a negative integer is \(\phantom{-}\).

\[= \phantom{-}\]

Multiply. The product of two negative integers is \(\phantom{-}\).

Currency  Use the table to find the average of the differences in the values of a Canadian dollar and a U.S. dollar for 1994–1997.

<table>
<thead>
<tr>
<th>Year</th>
<th>Canadian Dollar</th>
<th>U.S. Dollar</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>73</td>
<td>100</td>
<td>-27</td>
</tr>
<tr>
<td>1995</td>
<td>73</td>
<td>100</td>
<td>-27</td>
</tr>
<tr>
<td>1996</td>
<td>74</td>
<td>100</td>
<td>-26</td>
</tr>
<tr>
<td>1997</td>
<td>72</td>
<td>100</td>
<td>-28</td>
</tr>
</tbody>
</table>

For 1994 to 1997, the average difference was \(\phantom{-}\).

Quick Check

1. Patterns Use a pattern to simplify \(-3(-4)\).

\[\phantom{-}\]

2. Simplify each product.
   a. \(2(-6) = \phantom{-}\)
   b. \(4(-3) = \phantom{-}\)
   c. \(7(-2) = \phantom{-}\)
   d. \(-4 \cdot 8 (-2) = \phantom{-}\)
   e. \(6(-3)(5) = \phantom{-}\)
   f. \(-7 \cdot (-14) \cdot 0 = \phantom{-}\)

3. Simplify each quotient.
   a. \(-32 \div 8 = \phantom{-}\)
   b. \(-48 \div (-6) = \phantom{-}\)
   c. \(-56 \div (-4) = \phantom{-}\)
   d. Find the average of 4, -3, -5, 2, and -8.

\[\phantom{-}\]
Weather The temperature dropped 5 degrees each hour for 7 h. Use an integer to represent the total change in temperature.

Understand the Problem

1. How many degrees did the temperature drop each hour?

2. For how many hours did the temperature drop?

3. What are you asked to do?

Make and Carry Out a Plan

4. Use repeated subtraction to solve the problem. How many times will you subtract 5 degrees?

5. Use a number line to show your repeated subtraction. Continue on the number line below until you have subtracted \(-5\) the correct number of times.

\[ \begin{array}{ccccccccccc}
& & & & & & & & & & \\
\end{array} \]

6. What integer represents the total change in temperature?

Check the Answer

7. To check your answer, multiply the number of degrees the temperature dropped each hour by the number of hours. Your answer should be the same as your answer to Question 6.

Solve Another Problem

8. A scuba diver descends 10 ft every 10 seconds. Use an integer to represent the position of the diver after 1 min (60 seconds).
Practice 1-9

Use repeated addition, patterns, or rules to find each product or quotient.
1. 23 · 16
2. 8 · 7(−6)
3. −17 · 3
4. −24 ÷ 4
5. −65 ÷ 5
6. 117 ÷ (−1)
7. −30 ÷ (−6)
8. −21 ÷ (−3)
9. 63 ÷ (−21)
10. 5(−1)(−9)
11. −6(−3) · 2
12. −3 · 7(−2)
13. \(\frac{1.512}{−42}\)
14. \(\frac{−4.875}{−65}\)
15. \(\frac{−15(−3)}{−9}\)

Compare. Use >, <, or = to complete each statement.
16. −7(5) \(\square\) −6 · (−6)
17. −20 · (−5) \(\square\) 10 · |−10|
18. 3(−6) \(\square\) −3(6)
19. 121 ÷ (−11) \(\square\) −45 ÷ (−6)
20. −40 ÷ 8 \(\square\) 40 ÷ (−8)
21. −54 ÷ 9 \(\square\) 21 ÷ (−3)

For each group, find the average.
22. temperatures: 6°, −15°, −24°, 3°, −25°
23. bank balances: $52, −$7, $20, −$63, −$82
24. stock price changes: $6, −$6, −$9, $1, $3
25. golf scores: −2, 0, 3, −2, −3, 1, −4
26. elevations (ft): −120, 168, −60, −42, −36

Write a multiplication or division sentence to answer the question.
27. The temperature dropped 4° each hour for 3 hours. What was the total change in temperature?
Reteaching 1-9

Multiplying and Dividing Integers

Multiplying and dividing integers is very similar to multiplying and dividing whole numbers. Just remember the two basic rules for determining the sign of the product or quotient.

**Rule 1:** The product or quotient of two integers with the *same sign* is positive.

**Rule 2:** The product or quotient of two integers with *opposite signs* is negative.

**Find each product or quotient.**

<table>
<thead>
<tr>
<th></th>
<th>5 \cdot 7</th>
<th>(-2) \cdot (-3)</th>
<th>15 \div 3</th>
<th>-40 \div (-10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>5 \cdot 7 = 35</td>
<td>-2(-3) = 6</td>
<td>15 \div 3 = 5</td>
<td>-40 \div (-10) = 4</td>
</tr>
<tr>
<td>b</td>
<td>Same sign (both +)</td>
<td>Same sign (both +)</td>
<td>Same sign (both +)</td>
<td>Same sign (both +)</td>
</tr>
<tr>
<td>c</td>
<td>Same sign (both −)</td>
<td>Same sign (both −)</td>
<td>Same sign (both −)</td>
<td>Same sign (both −)</td>
</tr>
<tr>
<td>d</td>
<td>Opposite signs (−, +)</td>
<td>Opposite signs (−, +)</td>
<td>Opposite signs (−, +)</td>
<td>Opposite signs (−, +)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2 \cdot 7</th>
<th>-15 \div 3</th>
<th>40 \div (-10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>-5 \cdot 7 = -35</td>
<td>-15 \div 3 = -5</td>
<td>40 \div (-10) = -4</td>
</tr>
<tr>
<td>f</td>
<td>2(-3) = -6</td>
<td>2(-3) = -6</td>
<td>2(-3) = -6</td>
</tr>
<tr>
<td>g</td>
<td>Opposite signs (+, −)</td>
<td>Opposite signs (+, −)</td>
<td>Opposite signs (+, −)</td>
</tr>
<tr>
<td>h</td>
<td>Opposite signs (+, −)</td>
<td>Opposite signs (+, −)</td>
<td>Opposite signs (+, −)</td>
</tr>
</tbody>
</table>

**Complete the table. The first row has been done for you.**

<table>
<thead>
<tr>
<th></th>
<th>Same or Opposite Sign?</th>
<th>Sign of Product or Quotient</th>
<th>Product or Quotient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-91 \div (-13)</td>
<td>Opposite</td>
<td>Negative</td>
</tr>
<tr>
<td>2</td>
<td>6 \cdot 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>72 \div -9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-3(-6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-18 \div 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>11 \cdot (-5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>52 \div 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-12(6)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**A. Finger Multiplication**

Adding on your fingers is easy. Here is how to multiply numbers from 6 to 10 using your fingers.

**Example**

Multiply: \( 9 \times 8 \)

**Solution**

Imagine the fingers of both hands are numbered from 6 (thumb) to 10 (little finger). Touch finger 9 on one hand to finger 8 on the other hand. Bend any fingers beyond the touching fingers.

To find the **tens’ digit of the product**: Add the upright fingers.

\[ 4 + 3 = 7 \]

1 \times 2 = 2

To find the **ones’ digit of the product**: Multiply the number of bent fingers on one hand times the number of bent fingers on the other hand.

**Product**: 72

Use your fingers to multiply.

1. \( 7 \times 8 \)  
2. \( 9 \times 9 \)  
3. \( 9 \times 7 \)  
4. \( 6 \times 8 \)

**B. Binary Multiplication**

Using only multiplication and division by 2, you can find the product of any two numbers.

**Example**

Multiply: \( 39(-13) \)

**Solution**

Write the factors side by side.

\[ 39 \quad -13 \]

On the left side, divide by 2, dropping any remainder. On the right side multiply by 2.

\[ 19 \quad -26 \]

\[ 9 \quad -52 \]

Continue until you reach 1 on the left.

\[ 4 \quad -104 \]
\[ 2 \quad -208 \]
\[ 1 \quad -416 \]

List those numbers from the right side that are positioned across from the odd numbers on the left side. Add them.

\[ -13 \]
\[ -26 \]
\[ -52 \]
\[ -416 \]

Product: \(-507\)

Find each product using binary multiplication.

5. \( 22 \times 17 \)  
6. \( 45(-25) \)  
7. \( 68(-33) \)  
8. \( 75(-41) \)
Study Skill Plan your time, whether you are studying or taking a test. Look at the entire amount of time you have and divide it into portions that you allocate to each task. Keep track of whether you are on schedule.

Write an explanation in words for the meaning of each mathematical expression or statement.

1. $2 \div p$

2. $|x|$

3. $-10$

4. $y < -2$

5. $7a$

Write each expression or statement with math symbols.

6. the sum of $a$ and $b$

7. $3 \div 15$

8. $2 \times (x + y)$

9. The opposite of $m$ is less than 2.

10. $6 \less p$

11. the quotient of 12 and $t$

12. the absolute value of 3
Lesson 1-10

The Coordinate Plane

Lesson Objectives

- Name coordinates and quadrants in the coordinate plane
- Graph points in the coordinate plane

NAEP 2005 Strand: Algebra
Topic: Algebraic Representations

Local Standards: ________________________________

Vocabulary

A coordinate plane is ________________________________

The x-axis is ________________________________

The y-axis is ________________________________

Quadrants are ________________________________

The origin is ________________________________

An ordered pair is ________________________________

An x-coordinate is ________________________________

A y-coordinate is ________________________________

\[ P(\text{, }) \]

\[ O \text{ is the origin, } , \text{ where the axes intersect.} \]
Examples

1. **Naming Coordinates and Quadrants** Write the coordinates of point $G$. In which quadrant is point $G$ located?

Point $G$ is located units to the left of the $y$-axis. So the $x$-coordinate is . The point is units below the $x$-axis. So the $y$-coordinate is .

The coordinates of point $G$ are $(\underline{\quad}, \underline{\quad})$. Point $G$ is located in Quadrant $\underline{\quad}$.

2. **Graphing Points** Graph point $M(-3, 3)$.

- **Step 1**: Start at the origin.
- **Step 2**: Move units to the .
- **Step 3**: Move units up. Draw a dot. Label it .

Quick Check

1. a. Use the graph in Example 1. Write the coordinates of $E$ and $F$.

b. Identify the quadrants in which $E$ and $F$ are located.

2. Graph these points on one coordinate plane: $K(3, 1), L(-2, 1),$ and $M(-2, -4)$. Then describe the figure that is formed by connecting points $K, L,$ and $M$. 

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1-10 • Guided Problem Solving

GPS Student Page 55, Exercise 55

Geometry  

**PQRS** is a square. Find the coordinates of **S**.

\[ P(-5, 0), Q(0, 5), R(5, 0), S(7, 7) \]

**Understand the Problem**

1. What shape is **PQRS**? 

2. What information are you given about points **P**, **Q**, and **R**? 

3. What are you asked to find? 

**Make and Carry Out a Plan**

4. Graph **P**, **Q**, and **R** on the graph below.

5. Draw lines to connect **P** to **Q** and **Q** to **R**. These are two sides of the square.

6. What is true about the four sides of a square? 

7. Draw the two missing sides of the square.

8. **S** is the point where the two new sides meet. What are the coordinates of **S**? 

**Check the Answer**

9. To check your answer, use a ruler to measure each side. Since **PQRS** is a square, all four sides should have the same length.

**Solve Another Problem**

10. **JKLM** is a rectangle. Find the coordinates of **M**.

\[ J(-4, 2), K(-4, -2), L(4, -2), M(7, 7) \]
Practice 1-10

The Coordinate Plane

Graph each point.

1. \(A(-2, 2)\)  
2. \(B(0, 3)\)  
3. \(C(-3, 0)\)  
4. \(D(2, 3)\)  
5. \(E(-1, -2)\)  
6. \(F(4, -2)\)

Write the coordinates of each point.

7. \(A\)  
8. \(B\)  
9. \(C\)  
10. \(D\)

In which quadrant or on what axis does each point fall?

11. \(A\)  
12. \(B\)  
13. \(C\)  
14. \(D\)

Name the point with the given coordinates.

15. \((1, 4)\)  
16. \((-3, 0)\)  
17. \((5, -1)\)  
18. \((-2, -4)\)

Complete using positive, or negative, or zero.

19. In Quadrant II, \(x\) is \(\) and \(y\) is \(\).
20. In Quadrant III, \(x\) is \(\) and \(y\) is \(\).
21. On the \(y\)-axis \(x\) is \(\).
22. On the \(x\)-axis \(y\) is \(\).
Write the coordinates of point A.

Point A is 3 units to the right of the y-axis. So the x-coordinate is 3. It is 4 units below the x-axis. So the y-coordinate is $-4$. The coordinates of point A are $(3, -4)$.

In which quadrant is point A located?

Compare the point to the diagram. Point A is in the fourth quadrant.

Write the coordinates of each point.

1. $A \quad \quad \quad \quad 2. \quad B \quad \quad \quad$

3. $C \quad \quad \quad \quad 4. \quad D \quad \quad \quad$

5. $E \quad \quad \quad \quad 6. \quad F \quad \quad \quad$

7. $G \quad \quad \quad \quad 8. \quad H \quad \quad \quad$

In which quadrant does each point lie?

9. $A \quad \quad \quad \quad 10. \quad B \quad \quad \quad$

11. $C \quad \quad \quad \quad 12. \quad D \quad \quad \quad$

13. $E \quad \quad \quad \quad 14. \quad F \quad \quad \quad$

15. $G \quad \quad \quad \quad 16. \quad H \quad \quad \quad$
Enrichment 1-10

Latitude and Longitude

Geographers divide the earth into a coordinate grid using latitude and longitude lines. Latitude lines are parallel to the equator and run from 90°N (the North Pole) to 90°S (the South Pole). Longitude lines are measured east and west of the prime meridian, the 0° longitude line which runs through Greenwich, England. Point A in the figure has coordinates (15°S, 45°W).

Write the coordinates, giving the latitude first.

1. B ____________________________
2. C ____________________________
3. D ____________________________
4. E ____________________________
5. F ____________________________ 6. South Pole ______________________

Graph each point on the coordinate grid above. Write the letter beside the point.

10. J(0°, 30°E) 11. K(75°N, 82°E) 12. L(50°N, 55°W)

13. How many degrees of latitude separate Halifax, Nova Scotia (45°N, 65°W), and Cordoba, Argentina (32°S, 65°W)?

14. How many degrees of longitude separate Baku, U.S.S.R. (41°N, 50°E), and New Haven, CT (41°N, 73°W)?

15. One degree of longitude at the equator equals 69.2 mi. How far is it from Quito, Ecuador (0°, 79°W), to Kampala, Uganda (0°, 32°E)?

16. Belem, Brazil (0°, 48°W), is located due west of Libreville, Gabon. The distance between the cities is 3944.4 mi. Give the latitude and longitude of Libreville.
1D: Visual Vocabulary Practice

Study Skill  The Glossary contains the key vocabulary for this course.

Concept List

<table>
<thead>
<tr>
<th>opposites</th>
<th>ordered pair</th>
<th>origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>quadrants</td>
<td>variable</td>
<td>variable expression</td>
</tr>
<tr>
<td>x-axis</td>
<td>y-axis</td>
<td>y-coordinate</td>
</tr>
</tbody>
</table>

Write the concept that best describes each exercise. Choose from the concept list above.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The letter “c” in $24c + 8$</td>
<td></td>
</tr>
<tr>
<td>2. $10d - 3 + a$</td>
<td></td>
</tr>
<tr>
<td>3. $-9$ and $9$</td>
<td></td>
</tr>
<tr>
<td>4. (2, $-3$)</td>
<td></td>
</tr>
<tr>
<td>5. The number 8 in (5, 8)</td>
<td></td>
</tr>
</tbody>
</table>
1F: Vocabulary Review Puzzle

Study Skill  The language of mathematics has precise definitions for each vocabulary word or phrase. To help you learn these definitions, keep a list of the new words in each chapter, along with their definitions and an example.

Write the vocabulary word for each description. Complete the word search puzzle by finding and circling each vocabulary word. For help, use the glossary in your textbook. Remember that a word may go right to left, left to right, or it may go up as well as down.

1. a conclusion reached by observing patterns  
2. the plane formed by the intersection of two number lines  
3. a whole number or its opposite  
4. reasoning that makes conclusions based on patterns  
5. the horizontal or vertical number line on the coordinate plane  
6. a letter that stands for a number  
7. one of the four parts of the coordinate plane  
8. point of intersection for the axes  
9. kind of value that gives the distance of a number from zero  
10. replacing each variable with a number in an expression and simplifying the result

INTEGRATE
ENTERTAIN
ETERNITY
ENTER
TERMINATE
CART
ACCELERATE
CABLE
AXE

INTEGRATE
ENTERTAIN
ETERNITY
ENTER
TERMINATE
CART
ACCELERATE
CABLE
AXE